

Model of embedded spaces: the field equations

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Abstract

A study of the Model of Embedded Spaces (MES) with a relativistic version of Finslerian geometry is continued. The field equations of the MES (Einstein and Maxwell types) are derived, and this formally completes geometrization of classical electrodynamics. The minimal action principle leads to geometrization of the field sources (the right-hand sides of the equations) and, as a consequence, to a field hypothesis of matter, a direct confirmation of W. Clifford's ideas.

1 Introduction

Judging by the state of the art, the idea of treating physical phenomena as geometric ones in the really existing space-time of the Universe will be long attractive for researchers. It was put forward by W. Clifford at the end of the 19th century [1] and partly proved by H. Minkowski [2] and A. Einstein in collaboration with M. Grossmann [3] in the first quarter of the last century. The results of these works essentially changed the general notions on the substance of entire areas of physics. Geometrically, the essence of these works reduced to consecutively changing the space-time models of physics. Obviously, these were the first but uniquely effective steps toward geometrization of physics.

In our opinion, subsequent effort in this area did not lead to results comparable with those cited above. The stumbling block was the problem of geometrization of classical electrodynamics¹.

The main geometric idea behind the attempts of finding a solution to this problem (as well as the attempts of geometrization of weak and strong interactions) was to apply different types of more or less physically grounded generalizations of the Riemannian geometry, based, e.g., on the assumptions of sectional curvature [4], higher dimensions of real space-time [5, 6, 7], torsion [8] and so on. (Some new results were obtained in the framework of the relational approach — the works on the theory of physical structures [9, 10]. Based on their originally accepted principles, the authors deduced such realistic objects as metric space-time and physically acceptable generalizations of the Riemannian geometry, including versions with higher dimensions etc.)

To do justice to adherents of the “Riemannian” approach, we, however, should give some arguments. In our opinion, the most productive is only such a Riemannian generalization that is verified by experi-

¹This problem can be used as a test problem for

any attempts of geometrization of physics.

ment, though partly, on *all* scales (macroscopic, molar and microscopic) because the electromagnetic interaction is long-range. Besides, it is well known that the possibly existing higher dimensions have a Planck scale and must be compactified, which clearly contradicts the obvious first judgement².

Thus we may suggest that a progress in geometrization of physics is possible only on the way of *qualitative* modifications of the existing space-time model, which requires a geometry qualitatively more general than the Riemannian one. For example, a geometry in which Riemannian space is tangential to the space of the new model.

The Finslerian 4D model could be a real candidate for this modification [11]. This geometry allows for a clear physical interpretation: geometric properties of space may depend on the state of local classical matter (x^i, \dot{x}^k) , not only on the coordinates (as in the Riemannian model).

Although this model is not generally recognized in physics, it is used as a background for many works on geometrization of physical phenomena, including works devoted to some aspects of classical electrodynamics, e.g., [12]. There are also many generalizations of the Finslerian model, and the situation in this area, including the results, is similar to the “Riemannian” one.

Recently, the author managed to give a reason for such a situation with the Finslerian model. The point is that the classical Finslerian geometry (as well as its generalizations) is not relativistic [13]. It means that this geometry cannot play the role of the geometry of a **relativistic** generalization of the modern Riemannian model.

For Clifford’s hypothesis, this fact means a necessity of a) creating a relativistic Fins-

lerian geometry and b) proposing a new 4D space-time model which could be described by this geometry. Here, the problem of geometrizing electrodynamics should be considered as a test problem.

A new model of space-time³ and a simple metric version of the geometry were developed in [14, 15]. The final formal effort in realization of this programme, derivation of the field equations, is described in this paper.

It is clear that the required equations must be a set of local equations for the field potentials of MES: an equation of Einstein type (for the gravitational potential, the metric tensor g_{ik}) and an equation of Maxwell type (for the electromagnetic potential, Cartan’s torsion tensor $C_{i,kl}$). (The equations must be generally covariant, satisfy the correspondence principle and contain not higher than second-order derivatives.) In spite of the fact that the potential of the developed electromagnetic generalization is a third-order tensor, the prospective source in the Maxwell-type equation will be the vector of electric current density.

Besides, geometrization of fields requires a proper geometric understanding of the method used to find these equations (the minimal action principle in this case).

Thus, the aim of the work is derivation of MES field equations with, as far as possible, a clear interpretation of both the method and results obtained.

2 Lagrangian density and action

In our case, as in the construction of GR, it is absolutely natural to use the curvature of MES geometry \bar{R} to construct the field

²We do not deny the opportunity of higher dimensions in Riemannian or some other models, we only doubt whether geometrization of physical phenomena of any scales can be realized using a geometry of the Planck scale.

³The model of embedded spaces (MES).

Lagrangian density,

$$\bar{R} = g^{ik} \bar{R}_{ik}, \quad (1)$$

where the Ricci tensor is expressed in terms of the geometry connection as [15]

$$\bar{R}_{ik} = \frac{\partial a^l_{ik}}{\partial x^l} - \frac{\partial a^l_{il}}{\partial x^k} + a^l_{ik} a^m_{lm} - a^l_{im} a^m_{kl}.$$

Since the connection $a_{i,kl} = \Gamma_{i,kl} + \omega_{i,kl}$ is a sum of the Christoffel and Lorentz terms,

$$\begin{aligned} \omega_{i,kl} &= (F_{lm,ik} + F_{km,il} - F_{im,kl})u^m, \\ F_{ik,lm} &= \partial C_{k,lm}/\partial x^i - \partial C_{i,lm}/\partial x^k, \end{aligned} \quad (2)$$

where

$$2C_{i,kl} = \partial g_{kl}/\partial u^i \quad (3)$$

is Cartan's relativistic torsion tensor, the curvature (1) splits into the Riemannian curvature R (related to Γ_{kl}^i in the standard way) and the Lorentz r parts

$$\bar{R} = R + r.$$

In what follows, the Lorentz part of the curvature

$$r = 2(\omega^{i,k}{}_{[k;i]} + \omega^{i,k}{}_{[k}\omega^l{}_{l]i})$$

is conveniently expressed in terms of $F_{ik,lm}$, separating the divergence term

$$\begin{aligned} r &= 4(F^{i[k,l]}{}_{l}u_i)_{;k} + 2B_{ik}u^i u^k \\ &\equiv \frac{4}{\sqrt{-g}}(\sqrt{-g}F^{i[k,l]}{}_{l}u_i)_{;k} + 2B_{ik}u^i u^k, \end{aligned} \quad (4)$$

where the symmetric tensor B_{ik} is

$$B_{ik} = F^l_{(i,[m}{}^n(F_{l]k),n]}{}^m - 2F_{n]k),l]}{}^m).$$

(The brackets (\dots) or $[\dots]$ near indices mean, as usual, symmetrization or anti-symmetrization, respectively.)

It seems reasonable to use the scalar $\bar{R}_{ik}u^i u^k$ as a geometric invariant which can also be used for building the desired Lagrange density. However, this is not true:

the Ricci tensor \bar{R}_{ik} is a function of the connection, and the anisotropy of the MES has already been taken into account in the connection (2).

One of the basic assumptions of the MES is the concept of a congruence of curves (trajectories of the initial matter congruence), where the “initial” (or “bare”) matter means matter without contributions of its own fields to inertia. To be more specific, let us assume that this matter is distributed, with the densities of “bare” inertial mass and “bare” electric charge μ_0 and ρ_0 , respectively. Moreover, let the charge distribution be proportional to the mass distribution,

$$k = (\rho_0/\mu_0)^2, \quad (6)$$

where k is the gravitational constant.

At first sight, this assumption, unifying so strongly the initial matter, has no reasonable grounds. However, in what follows it will be demonstrated that the field hypothesis guarantees renormalization of ρ_0 and μ_0 to values characteristic of dressed matter (moreover, the assumption (6) allows the existence of neutral matter). Besides, the following argument can be adduced: there exists the Eulerian description of continuous matter, which is equivalent to the Lagrangian description. In the framework of this description, the velocity of matter u^i is treated as a local field $u^i(x^k)$. Therefore, formally, the MES geometry may be treated as a partial anisotropic case of Riemannian geometry.

Then, for the quantity $\rho_0/(\mu_0 c^2)$, in accordance with the general principle of relativity, legitimate are only such values which are combinations of the world constants (up to some numerical factor). The quantity $(\rho_0/\mu_0)^2$ and the gravitational constant k are equidimensional, so that the simplest relation is (6).

Then the combination $-\bar{R}/2\kappa$ (as in

GR), with

$$\varkappa = 8\pi k c^{-4}, \quad (7)$$

may be interpreted as the field Lagrangian 4-density (in any case, it contains isotropic terms quadratic with respect to Γ_{kl}^i for the gravitational field).

Naturally, this interpretation can also be extended to the Lorentz term r of the curvature, especially to the terms quadratic with respect to $F_{ik,lm}$. The fact that $F_{ik,lm}$ is included in r only as a contraction with u^k can be treated as a result of MES anisotropy.

The integral action of the physical system also contains the free initial matter term. Since this matter moves along geodesics of space, its Lagrangian density must be chosen as

$$\Lambda_0 = -\mu_0 c^2 ds / \sqrt{g_{00}} dx^0 = -ci_{(0)}u, \quad (8)$$

where

$$i_{(0)}^i = \mu_0 c dx^i / \sqrt{g_{00}} dx^0 \quad (9)$$

is the current density of inertial mass of the initial matter.

Thus the sought-for action must include the following terms:

$$S \sim -c^{-1} \int_{\Omega} (\Lambda_0 + \bar{R}/2\varkappa) \sqrt{-g} d\Omega, \quad (10)$$

where $d\Omega$ is the 4-volume element.

Further, to formulate the variation problem, we need to define its independent variables. At first sight, these should be the metric tensor g^{ik} (the gravitational field) and the tensor $C_{i,kl}$ (the electromagnetic field). However, this supposition is wrong since they *are not* independent quantities (see the definition (3)). Therefore, the following geometric approach to the problem is valid.

As a curvature criterion at some point of Riemannian space, we may use a scalar quantity, the interval $ds = \sqrt{g_{ik}(x^l) dx^i dx^k}$, which is the distance between this point

and an infinitely close point (with the coordinates $(x^i + dx^i)$). The interval is the length of a segment of some curve passing through these points, and the segment itself is situated along the unit tangential vector $u^i = dx^i/ds$ of the curve at the point (x^i) . The Hilbert variation with respect to the metric g_{ik} (with respect to the squared linear point density of the Riemannian space) means variation of this segment length for fixed projections dx^i .

The case of MES space is more general: because of its anisotropy, it is necessary to take into account the orientation of this segment relative to a preferential direction at the point (x^i) (relative to the curve of the MES congruence which passes through this point). By virtue of the isomorphism of MES curves $u^i \leftrightarrow u_{\text{mat}}^i$ [14, 15], the metric of MES space at the point (x^i) can be considered as $g^{ik}(x^l, u^m)$. Clearly, this conclusion is valid for both the functional (10) and the particular case in which our curve belongs to the congruence of MES curves (matter geodesics).

Thus such a generalization of the Hilbert variation to the case of MES space is quite natural: it has two independent variations.

These are the “old” functional variation of (10) with respect to g^{ik} and a “new” variation of (10) with respect to an *explicit* dependence on u_i , because an implicit direction dependence is taken into account by the first variation⁴.

Such a choice of variables for the variation procedure makes it necessary to consider the norms. It means that the present variation problem is a problem with constraints imposed on the functional. Then the latter must be written as

$$S = -c^{-1} \int_{\Omega} \left[\Lambda_0 + (R + r)/(2\varkappa) \right]$$

⁴This conclusion is confirmed by the form of r , see (4) and (5): in the case of a linear dependence of g_{ik} on u^l , the field tensor $F_{ik,lm}$ depends only on the coordinates!

$$+\lambda_1 g_{ik} g^{ik} + \lambda_2 u_i u^i \Big] \sqrt{-g} d\Omega, \quad (11)$$

where λ_1 and λ_2 are Lagrange multipliers.

3 Equation of Einstein type

As can be derived by varying (11) in g^{ik} subject to the condition that λ_1 and λ_2 are invariable constants in compliance with the Lagrange method,

$$0 = \int_{\Omega} \left[\left(R_{ik} - \frac{g_{ik}}{2} R - \varkappa t_{ik}^{(m)} + 4u_{(i} B_{k)l} u^l - g_{ik} B_{lm} u^l u^m \right) \delta g^{ik} + 2u^l u^m \delta B_{lm} \right] \sqrt{-g} d\Omega,$$

where

$$t_{ik}^{(m)} = t_{ik}^{(0)} - 2\lambda_2 u_i u_k + (2\lambda_1 + \lambda_2) g_{ik}, \quad (12)$$

and $t_{ik}^{(0)} = -\Lambda_0 u_i u_k$ is the energy-momentum tensor (EMT) of free initial matter.

Variation of the last term requires particular attention, but bearing in mind that

$$\delta F_{ik,lm} \sim \frac{\partial^2 \delta g_{lm}}{\partial x^i \partial u^k} = 0,$$

because $\delta g_{lm} = 0$, we can find the Einstein-type equation in the form

$$R_{ik} - \frac{g_{ik}}{2} R = \varkappa \left(t_{ik}^{(m)} + t_{iklm}^{(em)} u^l u^m \right), \quad (13)$$

where

$$t_{iklm}^{(em)} = -\frac{4}{\varkappa} \left(B_{iklm} + g_{l(i} B_{k)m} - \frac{g_{ik}}{4} B_{lm} \right) \quad (14)$$

is the EMT of the electromagnetic field and the tensor B_{iklm} is

$$B_{iklm} = F_{il,n} [{}^p F_{km,p} {}^n] + 2(F_{nl,ip} F_{m,k}^{[n} - F_{nl,ik} F_{m,p}^{[n} - F_{il,k} F_{nm,p}^{[n} + F_{l,n(i} F_{k)m,p}^{[n} - 2F_{(il,n} F_{m,pk]}^{[n}], \quad (15)$$

so that both B_{iklm} and $t_{iklm}^{(em)}$ are symmetric with respect to indices inside the first and second pairs of indices.

4 Equation of Maxwell type

It is derived as an extremum condition of (11) in the directions $\delta|_{u_i} S = 0$. The extremum is found from the explicit dependence of S on u_i because the implicit dependence was already taken into account in the Einstein-type equation. Thus we have

$$\int_{\Omega} [\delta r + 2\varkappa \delta(\Lambda_0 + \lambda_2 u_i u^i)] \sqrt{-g} d\Omega = 0.$$

Using the expression (4) for r , taking into account the commutativity of the operators $\partial/\partial u_i$ and $\partial/\partial x^k$,

$$\delta u_{i,k} = \delta u^l \frac{\partial^2 u_i}{\partial u_l \partial x^k} = \delta u^l \frac{\partial^2 u_i}{\partial x^k \partial u_l} = 0,$$

we obtain

$$\frac{2}{\sqrt{-g}} (\sqrt{-g} F^{i[k,l]})_{,k} + 2B^i{}_{k} u^k + \varkappa (\Lambda_0 + 2\lambda_2) u^i = 0.$$

But this equation has not a generally covariant form.

The requirement of general covariance can be satisfied if only this equation is a set of equations:

$$F^{ik,l}{}_{l;k} + 2B^i{}_{k} u^k = -\varkappa (\Lambda_0 + 2\lambda_2) u^i,$$

$$F^{il,k}{}_{l} = 0, \quad (16)$$

where $F^{ik,l}{}_{l;k} \equiv (\sqrt{-g} F^{ik,l}{}_{l})_{,k} / \sqrt{-g}$.

5 Interpretation of the equations

A physical meaning of the equations can be comprehended only after a concrete definition of the MES “vacuum” concept. Partially, this definition was discussed earlier [14]. Now, as a development of this notion, based on the ideas of continuum matter used in

this paper, it will be more reasonable to define the MES “vacuum” as space areas of distributed initial matter where $\rho_0 \rightarrow 0$ and $\mu_0 \rightarrow 0$. The ratio of these quantities must satisfy (7).

Hence the vacuum of MES must have the properties of homogeneous and extremely weakly charged inertial matter⁵, moving with the velocity u^i . The gravitational constant, more precisely, its algebraic square root with a certain (yet unknown) sign

$$\pm\sqrt{k} = \rho_0/\mu_0 \quad (17)$$

is the main *characteristic* of MES vacuum, like the Plank constant for the density of space points [15].

Such an approach allows us to treat the Universe as an area of space with some fixed sign⁶ in (17).

Note that if we adopt this hypothesis, then, as a model of a charged particle, we can choose a thermal vacuum fluctuation which has a long lifetime due to its own fields preventing its decay. The fluctuation may have the above ρ_0 and μ_0 densities, but the condition (17) for it holds true.

Thus in this model

$$\Lambda_0 \neq 0, \quad t_{ik}^{(0)} \neq 0, \quad i_{(0)}^i \neq 0.$$

Further we must understand how the initial matter is “dressed” with fields. In other words, since the role of the EMT of dressed matter is played by $t_{ik}^{(m)}$ (12),

$$t_{ik}^{(m)} = -(\Lambda_0 + 2\lambda_2) u_i u_k + (2\lambda_1 + \lambda_2) g_{ik}, \quad (18)$$

it is necessary to find a relationship of the Lagrange factors λ_1 and λ_2 with the field quantities. To do so, it is convenient to use

the Einstein-type equation in the following form:

$$t_{ik}^{(m)} = (R_{ik} - g_{ik}R/2)/\varkappa - t^{(em)}_{iklm} u^l u^m.$$

Contractions with g_{ik} and $u^i u^k$ lead to equations for the Lagrange factors:

$$\begin{aligned} 4\lambda_1 + \lambda_2 &= (\Lambda_0 - R/\varkappa - t^{(em)i}_{ikl} u^k u^l)/2, \\ 2\lambda_1 - \lambda_2 &= \Lambda_0 + (R_{ik} u^i u^k - R/2)/\varkappa \\ &\quad - t^{(em)}_{iklm} u^i u^k u^l u^m, \end{aligned}$$

whence it follows

$$\begin{aligned} 6\lambda_1 &= 3\Lambda_0/2 - R/\varkappa + (R_{ik}/\varkappa \\ &\quad - t^{(em)}_{iklm} u^l u^m - t^{(em)l}_{lik}/2) u^i u^k, \\ 6\lambda_2 &= -3\Lambda_0 + R/\varkappa - (4R_{ik}/\varkappa \\ &\quad - 4t^{(em)}_{iklm} u^l u^m + t^{(em)l}_{lik}) u^i u^k. \end{aligned} \quad (19)$$

The meaning of these formulae is clear: they describe geometrized matter.

Comparison of (19) with (18) for the EMT of dressed matter leads to a firm conclusion that the geometrized EMT of matter is independent of the Lagrange density of the initial (bare) matter Λ_0 (!), but is completely determined by the fields (gravitational and electromagnetic) and by the matter velocity field (congruence of MES curves). (After substitution of (19) into (18), Λ_0 vanishes from the coefficients before $u^i u^k$ and g_{ik} .)

This result is also true for the Maxwell-type equation (16).

Consequently, the initial (“bare”) matter concept is just a redundant though rather convenient hypothesis.

This conclusion should be interpreted as a direct proof of Clifford’s hypothesis for MES space: the MES field equations *do not include* non-geometrical quantities (except the constant k).

Turning back, let us analyze the results of abandoning the initial matter hypothesis.

⁵A comparison of q/m of some charged elementary particle with \sqrt{k} readily shows how weak is this electric property of vacuum. E.g., for the proton we have $\sqrt{k}/(q_p/m_p) \simeq 10^{-21}$.

⁶Indirectly, this definiteness of the sign is confirmed by the Universe asymmetry with respect to the content of matter and antimatter.

First of all, it means that the first term in the action S (11) of the “matter + field” physical system is unnecessary. It can be set to zero or excluded from the Lagrange density, and the last two terms of the action S (with Lagrange factors) describe the matter terms:

$$\begin{aligned} 6\lambda_1 &= -R/\varkappa + (R_{ik}/\varkappa \\ &- t_{iklm}^{(em)} u^l u^m - t^{(em)l}_{lik}/2) u^i u^k, \\ 6\lambda_2 &= R/\varkappa - (4R_{ik}/\varkappa \\ &- 4t_{iklm}^{(em)} u^l u^m + t^{(em)l}_{lik}) u^i u^k, \end{aligned} \quad (19')$$

which means that to solve the set of field equations (13) and (16), it is sufficient to know the vector field of matter velocities u^i and the value of gravitational constant.

Secondly, the minimal action principle has an almost geometric meaning (this meaning will be exactly geometric if it will be proved that the gravitational constant k has a geometrical origin).

Thirdly, the question of measurability of the scalar field Λ_0 (physically very nontrivial) is closed.

Fourthly, the concept of “matter” reduces to particle-like solutions of the fields equations (more precisely, to the areas of these solutions which have large curvature). In this case, the model of vacuum simply suggests that there exist areas of space with minimal (zero as a limit) curvature.

The only characteristic of the MES vacuum (in any case, for molar and macroscopic scales) is the gravitational constant k .

And finally, the MES congruence of curves should be interpreted as world lines of areas (points in the limit) with great curvature of particle-like solutions.

Conclusion: the disavowal of the initial matter hypothesis has found such a convincing geometric and physical justification that its further usage would be a mistake.

The form of the matter EMT (18) shows that this matter can be treated as a perfect

fluid,

$$t_{ik}^{(m)} = (\varepsilon + p)u^i u^k - p g_{ik}, \quad (20)$$

where ε and p are the energy density and pressure, respectively.

So formally, by virtue of ε and p measurability, equation (13) can be treated as a standard gravitational field equation with a source, which includes two terms. The first term is the matter EMT (20) and the second one is $t_{iklm}^{(em)} u^l u^m$.

Assuming that the quantities ε and p are known, their relationship with the Lagrange factors are easily found by comparing (20) with (18) at $\Lambda_0 = 0$:

$$2\lambda_2 = -(\varepsilon + p), \quad 2\lambda_1 + \lambda_2 = -p.$$

Let us substitute this result to the first Eq.(16) (at $\Lambda_0 = 0$):

$$F^{ik,l}_{l;k} + 2B^i_{k} u^k = \varkappa(\varepsilon + p)u^i.$$

In compliance with the vacuum model and Eq.(17), the tensor $F_{ik,lm}$ is related to the dimensional tensor $f_{ik,lm}$ ⁷ by

$$F_{ik,lm} = \pm c^{-2} \sqrt{k} f_{ik,lm}, \quad (21)$$

hence this equation can be rewritten as (see (7))

$$f^{ik,l}_{l;k} \pm 2c^{-2} \sqrt{k} b^i_k u^k = \pm 8\pi c^{-2} \sqrt{k} (\varepsilon + p) u^i, \quad (22)$$

where $b_{ik} \equiv B_{ik}/kc^{-4}$ and

$$b_{ik} = f^l_{(i,[m}{}^n (f_{lk),n]}{}^m - 2f_{n]k,l}{}^m). \quad (23)$$

Now the analogy between Eq.(22) and the equation of Maxwell's electrodynamics is obvious: the Maxwell field tensor f^{ik} corresponds to the contraction $f^{ik,l}_l$, and the electric current density j^i is represented by

$$j^i = \mp 2c^{-1} \sqrt{k} (\varepsilon + p) u^i, \quad (24)$$

⁷Clearly, the role of α [14] is now played by $\alpha_0 \equiv \pm c^{-2} \sqrt{k}$.

from which directly follows the expression for the charge density

$$\rho = \mp 2c^{-2} \sqrt{kg_{00}}(\varepsilon + p)u^0, \quad (25)$$

which can be named the field model of the electric charge. Obviously, the equation of state of neutral matter is

$$\varepsilon + p = 0.$$

It seems obvious that any change in the electric charge of a physical system leads to a change of its energy and pressure because a) this is related to variation of the matter density (the charge carriers are particles, and so variation of the system charge is a change in the number of its particles), i.e., variation of both ε and p ; b) charged massless particles are unknown and c) a change in the charge leads to a change in the system field, i.e., additional changes in the system energy density and pressure. Hence the charge density, the energy density and pressure should be locally interrelated. The only questionable point is that this relation is qualitatively similar to (25). Note that (24) and (25) are a classical (although relativistic) formulas, whereas the charge carriers used in the experiment are quantum objects. Therefore we cannot state that it is also valid in the quantum case. This will certainly require experimental verification.

Here we must make a remark. At first sight, if a model of the electric charge is a corollary of the field hypothesis, why, in this case, we cannot treat the nonlinearity of the first equation (16) (or (22)) as an additional term in the right-hand side, defining an electrical current density? After all, it is a contraction with velocity. The answer is simple: the current density, up to a *scalar* factor, must coincide with the velocity of charged matter (as it is defined in electrodynamics).

The second equation of the set (16)

$$f^{il,k}{}_l = 0$$

has no analogue in vectorial electrodynamics.

It can be demonstrated that the set (16) determines the tensor $F_{ik,lm}$ with completeness of Maxwell's electrodynamics.

First, for the antisymmetric pair of indices we have the first equation of the system, which has the classical structure of the Maxwell equation (its nonlinearity is inessential).

Second, for each fixed antisymmetric pair of indices, there are 10 components of the symmetric pair of tensor indices. For their determination, this equation gives only one condition. That is, to completely determine the tensor, one needs 9 more independent conditions, which are given by the second equation (16). Really, the tensor $F^{il,k}{}_l$ has no symmetries with respect to free indices. Therefore, this equation gives 16 additional conditions. However, only 9 of them are independent because the symmetric pair of indices of the tensor $F_{ik,lm}$ are the indices of the metric tensor. The latter should always satisfy 7 conditions for a choice of the reference frame (4 of them are the conditions of choosing a spatial point and the other 3 are the conditions of choosing the direction of motion at this point).

Finally, we must mention the identity

$$\partial F_{ik,mn}/\partial x^l + \partial F_{li,mn}/\partial x^k + \partial F_{kl,mn}/\partial x^i = 0,$$

which has a generally covariant form. This can be easily proved using the orthogonality property of the potential $C_{i,kl}$ and the connection $a_{i,kl}$ [14].

6 Discussion

Construction of the field equations formally means that the test problem of geometrization of classical electrodynamics has a solution in the framework of the 4D MES. This solution is a "rich" generalization of classic

electrodynamics and leads to some serious conclusions on such fundamental concepts as matter, electric charge, vacuum etc. At the same time, the field hypothesis of matter comes to the fore, leaving no other treatment of the latter. It is the author's strong belief that this result is the only logically admissible result of physical treatment of Clifford's idea.

Identification of the fields $F^{ik,l}_l$ with electromagnetic fields will hardly cause any doubt. But identification of the residuary 9 additional fields of the tensor $F_{ik,lm}$, e.g., with classical analogues of the Yang-Mills fields⁸, requires a separate investigation.

Of particular interest will be the results of MES investigations on nonclassical scales. They will possibly allow us to unravel the puzzles of the origin of the gravitational constant, the only physical constant of the theory. Similar results will probably be obtained for other fundamental constants (see, e.g., the considerations in [15] concerning Planck's constant). In any case, Clifford's hypothesis suggests a necessity of obtaining these answers.

Evidently, the predictions of the developed theory need experimental verification. An electromagnetic "redshift" experiment was discussed earlier, see, e.g., [14]. Now there appeared at least two new corollaries: a non-field contribution to the mass of matter particles is equal to zero, and the dependence of the charge density on the system energy density and pressure. These two predictions have a principal meaning for both the theory and Clifford's hypothesis.

⁸In the classical case, $F_{ik,lm}$ has no non-Abelian terms because $C_{i,kl}$ is an unobservable quantity. Therefore we speak about a "classical analogue". The quantum case of motion [15] reduces to the particle "scanning" of its trajectory $\varepsilon \neq 0$. Here $C_{i,kl}$ is already an observable quantity, and thus $F_{ik,lm}$ becomes non-Abelian. However, construction of a geometry for this case (as a generalization of the one used) is still an unresolved problem.

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